

# ME 4555 - Lecture 15 - Simulating transfer functions

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Today we look at two different ways of simulating an LTI system numerically in Matlab.

We will use this system as an example:

$$2\ddot{y} + 3\dot{y} + 10y = 4\dot{u} + 10u$$

Take Laplace transform (zero initial conditions):

$$(2s^2 + 3s + 10) Y(s) = (4s + 10) U(s)$$

→ transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{4s + 10}{2s^2 + 3s + 10}$$

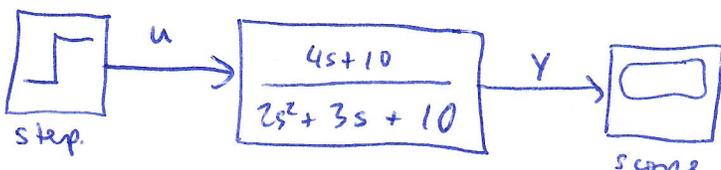
In Simulink, this is easy: use the "transfer fcn" (continuous)

block: Specify numerator coefficients: [4 10]

and denominator coefficients: [2 3 10]

Note! if a coeff is zero, i.e.,  $s^2 + 3$ , it's [1 0 3].

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 1s^2 + 0s + 3 \end{array}$$



Simulation in Matlab directly is also possible.

(2)

Step 1: create transfer function object. Two ways:

(a) directly:  $G = \text{tf}([4 \ 10], [2 \ 3 \ 10]);$

numerator coefficients      denominator coefficients.

(b) by building it from basic operations:

$$s = \text{tf}('s');$$

% special command, only works when used with 's'!

$$G = (4*s + 10) / (2*s^2 + 3*s + 10);$$

Transfer function objects can be manipulated using algebraic operations. (+, -, \*, /).

Step 2: simulate! Most commonly used functions are `impzle(G)` (plots impulse response) or `step(G)` (plots step response)

can also specify end time  $T$  (will plot from 0 to  $T$ )

using `impzle(G, T)` or `step(G, T)`.

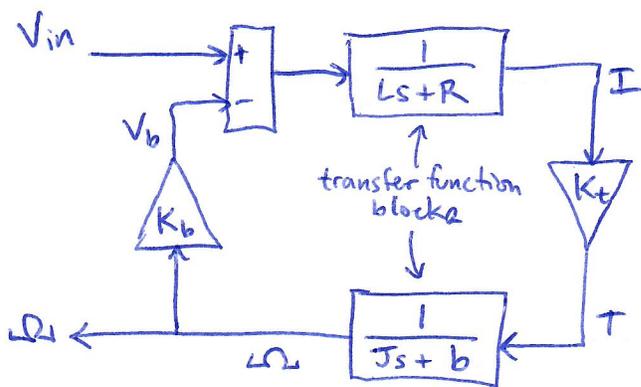


# Ex: DC motor simulation.

(4)

$$\text{ODEs: } \left\{ \begin{array}{l} L \frac{di}{dt} + Ri + V_b = V_{in} \\ T = K_t i \\ J\dot{\omega} + b\omega = T \\ V_b = K_b \omega \end{array} \right\} \xrightarrow{\mathcal{L}} \left\{ \begin{array}{l} (Ls + R)I + V_b = V_{in} \\ T = K_t I \\ (Js + b)\omega = T \\ V_b = K_b \omega \end{array} \right\}$$

write as transfer functions



$$\left\{ \begin{array}{l} \frac{I}{V_{in} - V_b} = \frac{1}{Ls + R} \\ \frac{T}{I} = K_t \\ \frac{\omega}{T} = \frac{1}{Js + b} \\ \frac{V_b}{\omega} = K_b \end{array} \right\}$$

Typical parameters for a small 12-volt motor:

- $J = 2.5 \times 10^{-4} \text{ Kg}\cdot\text{m}^2$
- $b = 1 \times 10^{-4} \text{ N}\cdot\text{m}\cdot\text{s}$
- $K_b = 0.05 \text{ V/rad/s}$
- $R = 0.5 \text{ }\Omega$
- $L = 1.5 \text{ mH}$
- $K_t = 0.05 \text{ N}\cdot\text{m/A}$

- ★ simulate motor. what is:
  - final speed (rad/s and rpm)?
  - how long does the motor take to get to its final speed?

- ★ use scope to observe current. Can you see the "switch-on surge" phenomenon? if you abruptly cut  $V_{in}$ , do you observe a "switch-off spike"?

- ★ can you alter motor parameters to produce oscillation?

- ★ what happens if weight is added to shaft, i.e.  $J$  is increased?

# Block diagrams with input derivatives

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If the system has derivatives of the input, how can we draw a block diagram using only integrators? Let's illustrate:

Ex:  $\ddot{x} + a_2 \dot{x} + a_1 x + a_0 x = b_2 \ddot{u} + b_1 \dot{u} + b_0 u$  (1)

The transfer function from  $u$  to  $x$  is:

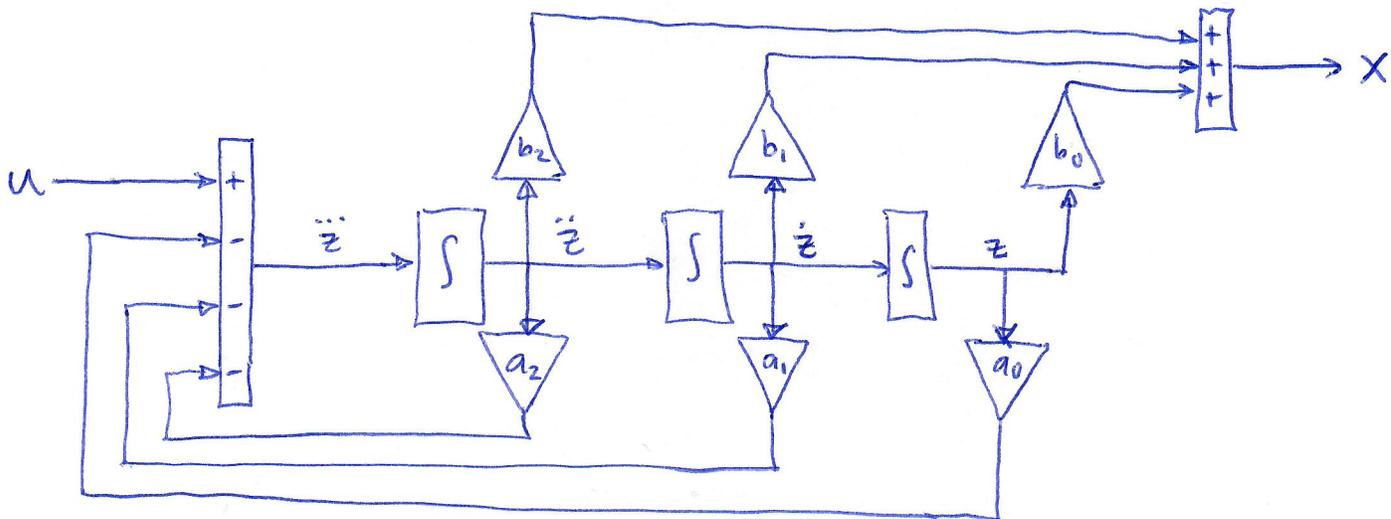
$$\frac{X}{U} = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

Define:  $Z = \frac{1}{s^3 + a_2 s^2 + a_1 s + a_0} U$ . Then,  $X = (b_2 s^2 + b_1 s + b_0) Z$ .

In other words, we can rewrite (1) as the following system:

$$\begin{cases} \ddot{z} + a_2 \dot{z} + a_1 z + a_0 z = U \\ b_2 \ddot{z} + b_1 \dot{z} + b_0 z = X \end{cases}$$

We can now draw the diagram  $u \rightarrow z$  in the usual way and then extract  $x$  because we already have all the ingredients!



# Block diagrams with input derivatives (alternative approach)

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Consider the same example:

$$\ddot{x} + a_2 \dot{x} + a_1 x + a_0 x = b_2 \ddot{u} + b_1 \dot{u} + b_0 u \quad (1)$$

We will define a sequence of auxiliary variables, starting with the output.

Define:  $z_3 := x$ . ODE becomes:  $\ddot{z}_3 + a_2 \dot{z}_3 + a_1 z_3 + a_0 z_3 = b_2 \ddot{u} + b_1 \dot{u} + b_0 u$ .

Gather highest derivative terms in the ODE, now define next variable:

Define:  $z_2 := \dot{z}_3 + a_2 z_3 - b_2 u$ . ODE becomes:  $\ddot{z}_2 + a_1 \dot{z}_2 + a_0 z_2 = b_1 \dot{u} + b_0 u$

Repeat the process, again gathering highest order terms.

Define:  $z_1 := \dot{z}_2 + a_1 z_2 - b_1 u$ . ODE becomes:  $\dot{z}_1 + a_0 z_1 = b_0 u$ .

Now that the ODE is 1<sup>st</sup> order, we can stop. Listing our equations in reverse order, we can see that (1) is equivalent to:

$$\begin{cases} \dot{z}_1 = b_0 u - a_0 z_1 & \text{(from ODE)} \\ \dot{z}_2 = z_1 + b_1 u - a_1 z_2 & \text{(from definition of } z_1) \\ \dot{z}_3 = z_2 + b_2 u - a_2 z_3 & \text{(from definition of } z_2) \\ x = z_3 & \text{(from definition of } z_3) \end{cases}$$

These 4 equations in 5 variables  $\{z_1, z_2, z_3, x, u\}$  are equivalent to (1)

We can now draw the block diagram  $u \rightarrow z_1 \rightarrow z_2 \rightarrow z_3 \rightarrow x$ :

